

Mean Field Theoretical Structure of He and Be Isotopes

S.J. Lee*, Y.W. Son, and J.W. Park

*Department of Physics and Institute of Natural Sciences,
Kyung Hee University, Suwon, KyungGiDo, Korea*

The structures of He and Be even-even isotopes are investigated using an axially symmetric Hartree-Fock approach with a Skyrme-III's mean field potential. In these simple HF calculations, He and Be isotopes appear to be prolate in their ground states and Be isotopes have oblate shape isomeric states. It is also shown that there exists a level crossing when the nuclear shape changes from the prolate state to the oblate state. The single neutron levels of Be isotopes exhibit a neutron magic number 6 instead of 8 and show that the level inversion between $1/2^-$ and $1/2^+$ levels occurs only for a largely deformed isotope. Protons are bound stronger in the isotope with more neutrons while neutron levels are somewhat insensitive to the number of neutrons and thus the nuclear size and also the neutron skin become larger as the neutron number increases. In these simple calculations with Skyrme-III's interaction no system with a clear indication of neutron halo was found among He and Be isotopes. Instead of it we have found $^8\text{He}+2n$, $2n+^8\text{He}+2n$, and $^{16}\text{Be}+2n$ like chain structures with clusters of two correlated neutrons. It is also shown that ^8He and ^{14}Be in their ground states are below the neutron drip line in which all nucleons are bound with negative energy and that ^{16}Be in its ground state is beyond the neutron drip line with two neutrons in positive energy levels.

Owing to the progress of experimental techniques, information concerning the ground state and the excited states of light unstable nuclei has rapidly increased. Furthermore radioactive ion beam accelerator will produce various unstable nuclei far from β -stability line. In light stable nuclei, it has already been known that clustering is one of the essential features of nuclear dynamics not only in excited states but also in ground states. Thus we may expect cluster structures in light unstable nuclei. Pioneering theoretical studies have suggested the development of cluster structures with a 2α core in Be and B isotopes [1, 2, 3, 4, 5]. The α chain structures of ^8Be and ^{12}C was investigated using a relativistic mean field (RMF) approximation [6]. In the case of ^{12}Be , the existence of cluster states was suggested in experimental measurements of the excited states [7] and in experiments of $^6\text{He}+^6\text{He}$ and $^8\text{He}+^4\text{He}$ breakup reactions [8]. The measured spin-parities of excited states indicate a rotational band with a large moment of inertia. These states are candidates of states with cluster structure. It is an interesting subject to investigate clustering aspects in Be isotopes. The cluster structures of ^{12}Be were studied with a potential model with He clusters [9] and with an algebraic version of resonating group model [10] using α particles as inert clusters. To eliminate the model assumption of inert α cluster an antisymmetrized molecular dynamics (AMD) [11] is used in study of Be isotopes. However this study uses inert Gaussian wave packets to represent nucleons. These studies treat α particles as inert clusters or nucleon as a fixed Gaussian packet, and thus miss any detailed structure of nuclei and any change of the internal structure of α particles in a nucleus. For a study of the detailed nuclear structure (such as size, shape, quadrupole moment, single nucleon levels) of these unstable nuclei, we need to use a self-consistent mean field approach in terms of nucleon itself. Mean field approaches have been applied successfully even for a small system such as nuclear ground states of ^6Be , ^7Be and ^7B [12, 13] and the Hartree-Fock (HF) energy of atomic He ground state [14]. RMF calculation with a nonlinear scalar field estimated binding energy of ^4He moderately well [6]. The ground states of nuclei with $2 \leq Z \leq 114$ around proton drip line have been studied using Skyrme HF+BCS [15]. Skyrme HF approach had described moderately well not only the ground state of ^4He but also the two- and four-particle excited states of ^4He [16].

An iterative method of intensity interferometry technique had been applied to dissociation of the three established two-neutron halo systems, ^6He , ^{11}Li and ^{14}Be , and the n-n correlation functions and corresponding source sizes had thus been extracted [17]. Based on the transverse momentum distributions after break-up of ^{11}Li into $^9\text{Li}+n+n$, it was suggested that the basic structure of ^{11}Li is a ^9Li -core surrounded by an extended cloud of two correlated neutrons [18, 19] at a large distance from the core. It was pointed out [20] that the extra stability provided by correlations between many neutrons surrounding a nuclear core may result in stable, exotic structures along or perhaps even beyond the neutron drip line. The ground state and low-lying states of ^{12}Be presented us with other subjects concerning the vanishing of a neutron magic number 8 [11]. The vanishing of the neutron magic number 8 was already known in a neighboring nucleus, ^{11}Be . The vanishing in ^{12}Be was predicted in the early theoretical work [21] using shell model configuration mixing. Contribution from $^{10}\text{Be} \otimes (sd)^2$ in the low-lying states of ^{12}Be were experimentally studied

* Email: ssjlee@khu.ac.kr

TABLE I: The binding energy per nucleon E_B (MeV) with CM correction [16], the root mean square radius R (fm), and the intrinsic quadrupole moment Q (fm²) of the lowest state of He isotopes.

	⁴ He	⁶ He	⁸ He	¹⁰ He	¹² He
E_B	7.15	5.13	4.30	3.04	2.27
$E_B(\text{exp})$ [32]	7.07	4.88	3.93	3.03	
R_{proton}	1.99	1.99	1.95	1.98	1.96
R_{neutron}	1.98	2.79	2.86	8.12	9.92
R_{total}	1.98	2.55	2.67	7.32	9.09
Q_{proton}	0.03	0.72	0.13	0.25	0.11
Q_{neutron}	0.03	5.56	0.68	223.11	439.88
Q_{total}	0.06	6.28	0.81	223.36	439.99

by ¹⁰Be(t,p)¹²Be reaction [22]. To study exotic nuclear structure without using configuration mixing we will use non-spherical axially symmetric mean field theory here.

In a nuclear mean field theory, a nuclear system is composed of nucleons interacting strongly through a mean field potential. While a nonrelativistic mean field approach uses a phenomenological potentials as a function of nucleon density, the mean field potential in a relativistic treatment is determined as a resultant of the meson exchange. After Walecka proposed a relativistic mean field (RMF) approach for a nuclear system in which Dirac nucleons interact by exchanging classical meson fields [23], the theory has been extended to include various quantum effects and successfully applied in describing nuclear matter and finite nuclei [24, 25, 26, 27, 28]. However the usage of the relativistic mean field approach is mostly limited to spherical or moderately deformed nuclei since the RMF failed in describing highly deformed nuclei unless using a nonlinear meson field [24, 25, 26]. On the other hand the nonrelativistic Hartree-Fock (HF) approach using empirical Skyrme interaction can describe highly deformed nuclei moderately well [29, 30] as well as describing spherical nuclei. Thus we use nonrelativistic HF approach here in describing structures of He and Be isotopes. As a first step, a simple HF method is used in this paper to investigate various properties of the lowest energy states of He and Be even-even isotopes before studying various excited states.

In a nonrelativistic HF approach, nucleon-nucleon nonlocal interactions are replaced by a density dependent Skyrme mean field potential which is a function of nuclear density and charge density in the nucleus. The level of each nucleon in a nucleus is determined by Schrödinger equation in the mean field potential. By occupying nucleons in these single nucleon levels, we determine the nuclear and charge densities in the nucleus which are then used to determine the new Skyrme mean field potential. Then this new potential is used to determine new levels of nucleon. We repeat this procedure until we get the converged results. By occupying Z single proton levels and N single neutron levels starting from the lowest level, we can obtain the lowest energy state (either ground state or an excited local minimum in shape variation) of a nucleus with Z protons and N neutrons. To discriminate from the particle-hole type excitation, we will call the excited local minimum state by shape isomeric state here since the state would live longer than the particle-hole type excited state. The numerical method and the Skyrme-IIIls interaction used here are exactly the same as that used in Ref.[30] which is applicable to a nuclear system with axial symmetry. The axially symmetric two dimensional space is denoted here by the body fixed symmetry axis z and the radius r from the symmetry z -axis. The level of each nucleon is identified by (Ω, π_z, π_r) where Ω is the total magnetic quantum number of the single nucleon level in the intrinsic z -direction and π_z and π_r are the z - and r -parity in this z - r space. While Ω and π_r stay to be a good quantum number during the iterative calculation due to the axial symmetry, π_z does not need to remain as a good quantum number unless we further restrict the system to have a reflection symmetry in z -direction too.

The results are summarized in Table I and II for He isotopes and in Table III and IV for Be isotopes. The binding energies obtained here with Skyrme-IIIls interaction were overestimated than the corresponding empirical values [6, 31, 32]. However the CM energy correction to the binding energy for these nuclei are very large (more than one third of binding energy for He isotopes and more than one fifth for Be) and thus the binding energies have large uncertainty here due to the approximate treatment of CM energy. Here the CM energy corrections were approximated by $1/A$ of total kinetic energy following the prescription used in Ref.[16]. Approximating CM energy correction using harmonic oscillator energy [33] makes the better agreement of binding energies with empirical values for Be isotopes but it makes the binding energies too large for He isotopes. Thus we examine here only the qualitative behavior of isotopes depending on the neutron number. For a quantitative examination we first need to implement CM energy

TABLE II: The level energy in MeV of single nucleon in level (Ω, π_z, π_r) of He isotopes in their ground state. Each level is twofold degenerated with $\pm\Omega$.

	(Ω, π_z, π_r)	^4He	^6He	^8He	^{10}He	^{12}He
proton	(0.5,+,+)	-14.41	-22.99	-31.51	-31.54	-31.91
	(0.5,-,+)	0.60	-9.70	-14.85	-14.89	-15.20
	(1.5,+,-)	0.54	-4.57	-13.86	-13.89	-14.43
neutron	(0.5,+,+)	-15.37	-16.28	-17.98	-17.96	-17.96
	(0.5,-,+)	-0.18	-1.96	-2.78	-2.80	-2.84
	(1.5,+,-)	-0.23	-0.84	-2.28	-2.30	-2.39
	(0.5,+,+)	1.62	0.77	0.77	0.71	0.67
	(0.5,-,+)	1.93	0.86	0.88	0.78	0.74

TABLE III: The binding energy per nucleon E_B (MeV) with CM correction [16], the root mean square radius R (fm), and the intrinsic quadrupole moment Q (fm²) of the ground state and the shape isomeric excited state of Be isotopes.

	^8Be	^8Be	^{10}Be	^{10}Be	^{12}Be	^{12}Be	^{14}Be	^{16}Be	^{16}Be	^{18}Be
E_B	6.71	6.04	6.72	6.51	6.04	6.03	5.25	4.49	4.25	3.83
$E_B(\text{exp})$ [32]	7.06		6.50		5.72		4.99	4.27		
R_{proton}	2.55	2.40	2.38	2.33	2.35	2.34	2.50	2.56	2.41	2.56
R_{neutron}	2.53	2.38	2.55	2.51	2.76	2.75	3.17	3.42	3.97	7.01
R_{total}	2.54	2.39	2.48	2.44	2.63	2.62	3.00	3.23	3.65	6.30
Q_{proton}	7.17	-2.65	4.41	-2.24	2.50	-1.75	5.93	6.64	-2.63	6.67
Q_{neutron}	7.07	-2.60	4.45	-2.86	1.66	-1.05	16.97	23.44	-9.70	287.76
Q_{total}	14.25	-5.25	8.86	-5.10	4.16	-2.80	22.90	30.08	-12.33	294.43

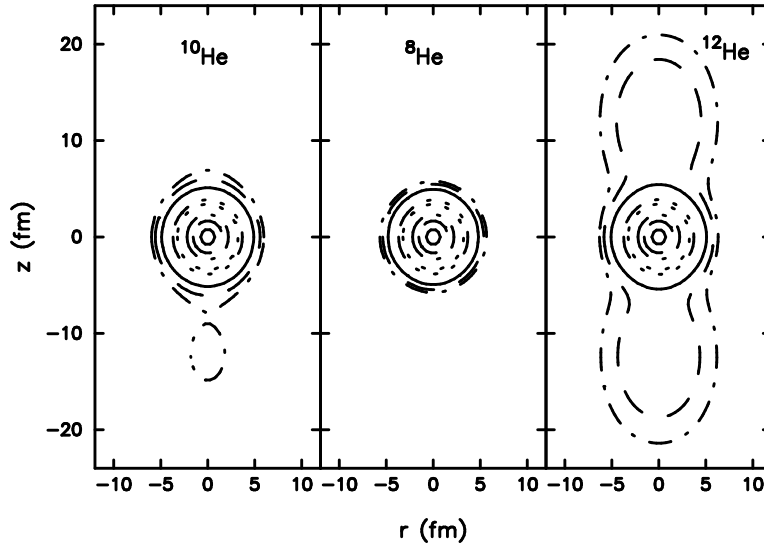
correction more self-consistently [34, 35] and need to search appropriate Skyrme interaction which predicts correctly not only the binding energy but also other properties too such as the size and deformation.

The binding energy and the root mean square (rms) radius of nuclear distribution show that ^{10}Be is bound most strongly among Be isotopes considered here while ^4He is most stable among He isotopes. Empirically, ^8Be is bound stronger than ^{10}Be . However AMD-HF calculation [12] also shows that ^{10}Be is bound stronger than ^8Be . The rms radius of proton distribution is smallest for ^8He among He isotope and for ^{12}Be among the ground states of Be isotopes. Of course the rms radius of neutron distribution is larger for the isotope with more neutrons. The values of the quadrupole moment show that the ground states of He and Be isotopes are prolate while the shape isomeric excited states of Be isotopes are oblate. Even though we have not required the reflection symmetry in z -direction here, the results show that the states of He and Be nuclei obtained here have a reflection symmetry in the z -direction (i.e.,

TABLE IV: The level energy in MeV of single nucleon in level (Ω, π_z, π_r) of Be isotopes in their ground state and shape isomeric state. Each level is twofold degenerated with $\pm\Omega$. Note here that the proton level (1.5, +, -) is occupied instead of the lower level (0.5, -, +) in the oblate ^{12}Be .

	(Ω, π_z, π_r)	^8Be	^8Be	^{10}Be	^{10}Be	^{12}Be	^{12}Be	^{14}Be	^{16}Be	^{16}Be	^{18}Be
proton	(0.5,+,+)	-21.41	-23.49	-30.29	-31.01	-35.87	-36.21	-38.22	-41.23	-40.55	-41.34
	(0.5,-,+)	-10.76	-6.96	-15.32	-14.05	-19.57	-20.14	-24.95	-29.41	-24.48	-29.53
	(1.5,+,-)	-3.53	-7.87	-12.71	-14.48	-19.55	-19.26	-20.85	-23.68	-25.28	-23.80
neutron	(0.5,+,+)	-23.25	-25.41	-25.29	-25.68	-25.11	-25.18	-24.50	-24.90	-25.52	-24.92
	(0.5,-,+)	-12.47	-8.72	-12.21	-9.17	-11.31	-9.43	-13.65	-14.76	-10.53	-14.76
	(1.5,+,-)	-5.20	-9.62	-7.18	-10.11	-8.58	-10.40	-7.64	-7.88	-11.69	-7.92
	(0.5,-,+)	-0.61	-2.34	-1.86	-2.70	-3.48	-3.67	-2.67	-3.07	-4.23	-3.11
	(0.5,+,+)	-1.54	0.81	-0.48	0.63	0.30	0.56	-2.15	-3.43	0.03	-3.44
	(0.5,-,+)	1.07	1.44	0.89	0.94	0.85	0.87	0.89	0.83	0.89	0.78
	(0.5,+,+)	1.04	2.01(1.5+-)	0.98	1.32	1.12	1.12	0.92	0.82	0.90	0.76
	(0.5,-,+)	1.46(0.5++)	1.98	1.48	1.52	1.37(0.5++)	1.49	1.03(1.5--)	0.30(1.5--)	0.45(2.5++)	0.27(1.5--)

FIG. 1: Contour plot of nuclear density distribution in He isotopes. The densities for each contour curves are 0.15, 0.10, 0.05, 0.01, 0.005, 0.001, 0.0005, and 0.00035 fm⁻³ from inside of the contour.



π_z is a good quantum number) and thus have a positive parity due to the filling of both $\pm\Omega$ levels of the even-even isotopes except for ^{10}He and ^{18}Be which have an asymmetric chain structure.

The single neutron levels in He and Be isotopes are somewhat insensitive to the number of neutrons in the isotope. Thus occupying the neutron levels from inner shell to outer shell makes the size of neutron distribution and the neutron skin larger for isotope with more neutrons. In contrast to this behavior of neutrons in He and Be isotopes, the single proton levels are very sensitive to the number of neutrons. The protons are bound stronger in the isotope with more neutrons up to ^8He and up to ^{16}Be . This sensitivity of proton levels cannot be examined by any models assuming an inert core or α cluster. AMD-HF calculations [12] also show a similar behavior of the single particle energies of protons and neutrons in ^8Be and ^{10}Be . In the oblate state of Be isotopes, the order of the single nucleon levels $(\Omega, \pi_z, \pi_r) = (1.5, +, -)$ and $(0.5, -, +)$ of proton and of neutron is reversed from the order in the prolate state. In a spherical limit, both of these two levels belong to the same $1p_{3/2}$ shell with different Ω values. In the oblate excited state of Be isotopes, the level $(1.5, +, -)$ of proton and of neutron is lower than the level $(0.5, -, +)$ of proton and of neutron respectively and thus two protons occupy the level $(1.5, +, -)$ making the nuclei to be oblate. On the other hand, in the prolate ground state of He and Be isotopes, the level $(0.5, -, +)$ is lower than the level $(1.5, +, -)$ and thus two protons occupy the level $(0.5, -, +)$ making the nuclei to be prolate. This exhibits the existence of a level crossing when the nuclear shape changes from the prolate state to the oblate state.

Even though it is insensitive to the number of neutrons, the rms radius of proton distribution shows that protons are most strongly bound in ^8He among He isotopes considered here while the rms radius of neutron distribution increases as the number of neutrons increases. Proton distribution in He isotopes is spherical (^4He) or near spherical. While the neutron distribution is spherical in ^4He and near spherical in ^8He , it is deformed prolate in ^6He . The nuclear density profiles (see Fig.1) show that the large rms radius and quadrupole moment of neutron distribution for ^{10}He and ^{12}He come from the $^8\text{He}+2n$ and $2n+^8\text{He}+2n$ like chain structure respectively where the centers of each cluster are about 6 fm apart. Note that in Fig.1 for ^{10}He we have a parity violating intrinsic state consisting of a ^8Be core and two neutrons mostly on one side, i.e., showing an approximated $^8\text{He}+2n$ like chain structure. On the other hand ^{12}He exhibits more exact $2n+^8\text{He}+2n$ like chain structure with two $2n$ clusters located symmetrically. The neutrons in the correlated two neutron clusters are in the single nucleon levels with positive energy (see Table II) which means that they are unbound to the ^8He cluster. This fact is consistent with the empirical neutron separation energy [32]; ^{10}He has a negative two-neutron separation energy of -1.07 MeV and a small positive one-neutron separation energy of 0.2 MeV. ^{10}He and ^{12}He may be either a resonance system or a Borromean system [36] (c.f., the one-neutron separation energy of ^9He is -1.27 MeV). Marqu  s et al. [17] used two-neutron interferometry as a probe of the neutron halo. However correlated neutrons can also be originated from a dineutron cluster or from a neutron skin which has mostly neutrons and is not necessarily a neutron halo.

The single proton levels are very sensitive to the number of neutrons up to ^8He while the single neutron levels are somewhat insensitive. The single nucleon levels in ^{10}He and ^{12}He stay similar to the levels in ^8He . The highest occupied neutron level is more strongly bound in ^8He (2.28 MeV) than in ^6He (1.96 MeV) and thus the neutron separation energy is larger for ^8He than for ^6He . Empirically, one-neutron separation energies are 1.86 MeV and 2.57

MeV for ${}^6\text{He}$ and ${}^8\text{He}$ respectively while two-neutron separation energies are 0.97 MeV and 2.14 MeV respectively [32, 37]. ${}^6\text{He}$ has also a large neutron rms radius which is similar to the rms radius of ${}^8\text{He}$. This large neutron rms radius and small neutron separation energy may indicate ${}^6\text{He}$ is a halo nucleus. However the large rms radius may come from a large deformation rather than from a halo structure. The quadrupole moment of ${}^6\text{He}$ is much larger than the quadrupole moments of ${}^4\text{He}$ and ${}^8\text{He}$. On the other hand, ${}^8\text{He}$, which is nearly spherical, has a larger neutron skin ($\Delta R = R_{\text{neutron}} - R_{\text{proton}} = 0.91$ fm) than ${}^6\text{He}$ and ${}^{16}\text{Be}$ ground state and it has a larger nuclear rms radius than ${}^{12}\text{Be}$. The proton rms radius of ${}^8\text{He}$ is the smallest among He isotopes and the nuclear quadrupole moment of ${}^8\text{He}$ is smaller than of ${}^6\text{He}$. However ${}^8\text{He}$ has a larger neutron separation energy than ${}^6\text{He}$ and the highest neutron level of ${}^8\text{He}$ has angular momentum of at least $l = 1$ ($\Omega = 1.5$ for this neutron level). Thus more detailed analysis are required to check if the ground state of ${}^8\text{He}$ obtained here is actually a halo nucleus.

Be isotopes are prolate in their ground states and they have oblate shape excited states in which the nucleons filled self-consistently from the lowest single nucleon levels. According to the binding energy per nucleon and the rms nuclear radius ${}^{10}\text{Be}$ is most strongly bound among the Be isotopes. This suggests the shell closure at a neutron magic number 6 instead of 8. The vanishing of the neutron magic number 8 was predicted in Refs.[11, 21, 22] by studying the ground state and low-lying states of ${}^{12}\text{Be}$. The appearance of a new magic number can be seen more directly from the single particle levels. There is a larger gap between the third (1.5, +, -) and the fourth (0.5, -, +) single neutron levels than the gap between the fourth (0.5, -, +) and the fifth (0.5, +, +) levels in He and Be isotopes (see Tables II and IV) independent of which proton levels are occupied. For a spherical or near spherical nuclei, this corresponds to a larger gap between $1p_{3/2}$ and $1p_{1/2}$ shells than the gap between $1p_{1/2}$ and $2s_{1/2}$ or $1d_{5/2}$ shells. It was suggested that the origin of the new magic number may be due to neutron halo formation [38]. But our calculation shows that the new magic number originates from a nuclear deformation. The gap between the fourth level (0.5, -, +) and the fifth level (0.5, +, +) is smaller for an isotope with a larger deformation, especially, of proton distribution. For a largely deformed nuclei, ${}^8\text{Be}$ and ${}^{16}\text{Be}$, a level inversion between the fifth and the fourth single particle levels occurs. Thus the intruder state $1/2^+$ causing the inversion between $1/2^+$ and $1/2^-$ for the ground state of ${}^{11}\text{Be}$ and ${}^9\text{He}$ [39, 40] may be originated from a large deformation of proton distribution independent of which levels of proton and neutron are occupied. It was also shown [41] that low-lying intruder states of ${}^8\text{Be}$, ${}^{10}\text{Be}$, and ${}^{12}\text{C}$ can be described better with a deformed oscillator model than with a spherical shell model.

The ground state of ${}^{12}\text{Be}$ has the smallest proton rms radius and the most spherical nuclear distribution among Be isotopes in its ground state. The density distribution shows that ${}^{12}\text{Be}$ has a small peanut shell shaped core of higher central density surrounded by a large spherical neutron skin of lower density without showing any increase of neutron density in radius. The proton levels (Ω, π_z, π_r) = (0.5, -, +) and (1.5, +, -) in the ground state of ${}^{12}\text{Be}$ are nearly degenerated and thus the proton shell is unclosed in this calculation with Skyrme-III interaction. In the spherical limit, both of these two levels belong to the $1p_{3/2}$ level and thus are exactly degenerated. This degeneracy causes the excited oblate state to be nearly degenerated with the prolate ground state for ${}^{12}\text{Be}$ (see Table III and discussion below). The ground state of ${}^8\text{Be}$ has a large quadrupole moment with a peanut shell (with two kernels) shaped nuclear distribution which may be considered as a two- α chain structure. The nuclear shape of ${}^{10}\text{Be}$ and ${}^{14}\text{Be}$ are deformed prolate but ${}^{10}\text{Be}$ has a small peanut shell shaped core. ${}^{16}\text{Be}$ has a peanut shell shaped nuclear distribution and ${}^{18}\text{Be}$ has an approximated ${}^{16}\text{Be}+2n$ like chain structure. The neutron rms radius of Be isotopes increases as the number of neutrons increases. The large neutron rms radius and quadrupole moment of ${}^{18}\text{Be}$ come from the ${}^{16}\text{Be}+2n$ like chain structure. The single nucleon levels in ${}^{18}\text{Be}$ stay similar to the levels in ${}^{16}\text{Be}$. Two neutrons occupying the highest single particle level (1.5, -, -) in ${}^{16}\text{Be}$ have a small positive energy but without showing any increase of neutron density in the large neutron skin. Thus, ${}^{16}\text{Be}$ is beyond the neutron drip line while ${}^{14}\text{Be}$ is inside of the drip line in these calculations. Jensen and Riisager [20] pointed out the possible existence of exotic nuclei even beyond the neutron drip line due to a correlation between many neutrons. Empirically, ${}^{16}\text{Be}$ has a negative two-neutron separation energy of -1.58 MeV and a small positive one-neutron separation energy of 0.19 MeV [32]. In ${}^{18}\text{Be}$ four neutrons occupy single neutron levels with positive energy in the nucleus. The ground states of ${}^{16}\text{Be}$ and ${}^{18}\text{Be}$ with neutrons in positive energy level may be either one of Borromean nucleus [36] or one of resonance. To check this, more detailed investigations are needed.

Be isotopes in their excited states have smaller rms radii and smaller magnitude of quadrupole moments than in their ground states. A four-body cluster model based on a resonating group model [39] also predicted the 2_1^+ excited state of ${}^{10}\text{Be}$ has smaller rms radius than the 0^+ ground state. The excitation energies are 5.4, 2.1, 0.1, and 3.8 MeV for ${}^8\text{Be}$, ${}^{10}\text{Be}$, ${}^{12}\text{Be}$, and ${}^{16}\text{Be}$ respectively. However, since we worked with non-spherical quantum numbers with axial symmetry, we cannot directly compare the excited states obtained here for Be isotopes with experimental results which are usually identified by spherical quantum numbers. The nearly degenerated (0.1 MeV excitation) oblate and prolate states of ${}^{12}\text{Be}$ may cause a shape coexistence [35] for the ground state of ${}^{12}\text{Be}$.

The lowest single nucleon level in each isotope is more strongly bound in the oblate excited state than in the prolate ground state while the next nucleon level is higher in the oblate shape isomeric state than in the prolate ground state. The third nucleon level of each isotope is also more strongly bound in the oblate excited state than in the prolate

ground state. Thus the energy gap between the second level $(\Omega, \pi_z, \pi_r) = (0.5, -, +)$ and the third level $(1.5, +, -)$ of proton and neutron in Be isotopes is larger in their prolate ground state (more than 2.5 MeV) than in their oblate shape isomeric state (less than about 1 MeV) except for the proton levels in ^{12}Be . This result is consistent with the fact that these two levels belong to a spherical $1p_{3/2}$ shell and the Be isotopes in their oblate excited states are more spherical than in their prolate ground states. In ^{12}Be with Skyrme-IIIls interaction, these two proton levels are nearly degenerated with the gap of 0.02 MeV in the prolate ground state while the gap is 0.88 MeV in the oblate excited state. The order of these single nucleon levels $(0.5, -, +)$ and $(1.5, +, -)$ in the excited oblate state of Be isotopes are reversed from the order in the prolate ground state except for proton levels in ^{12}Be . In the excited oblate state of ^{12}Be , the unoccupied $(0.5, -, +)$ proton level is lower than the occupied $(1.5, +, -)$ proton level. The protons in the excited oblate state of ^{12}Be is not filled self-consistently from the lowest single proton levels but is filled in a fixed configuration in this calculation with Skyrme-IIIls interaction. Thus the excited oblate state of ^{12}Be obtained here is a proton $2p - 2h$ excited state instead of a shape isomeric state. All other states of He and Be isotopes in these calculations are formed by filling protons and neutrons self-consistently from the lowest single nucleon levels.

While the same neutron levels of Be isotopes are occupied both in the prolate ground state and in the oblate excited state except for ^8Be , different proton levels are occupied in the ground state and in the excited state. The Be isotopes become prolate by filling the $(0.5, -, +)$ proton level and become oblate by filling the $(1.5, +, -)$ proton level. ^{12}Be is the most spherical among Be isotopes both in their ground states and in their excited states. The excited state of ^{16}Be has a large neutron rms radius (3.97 fm) and a large neutron skin ($\Delta R = 1.56$ fm) with relatively small quadrupole moment and four neutrons in positive energy levels. This probably indicates the excited oblate ^{16}Be is a halo nucleus. However more detailed analysis are required to check if the excited oblate state of ^{16}Be obtained here is actually a halo nucleus.

In conclusion of this paper we have investigated the mean field structure of He and Be isotopes in a nonrelativistic Hartree-Fock approximation with Skyrme-IIIls interaction in an axially symmetric configuration space. The ground and shape isomeric states of these isotopes are obtained by occupying protons and neutrons self-consistently from the lowest single nucleon levels. It is shown that the ground states of He and Be isotopes are prolate while the excited shape isomeric states of Be isotopes are oblate. There is a level crossing between the second and the third nucleon levels, $(\Omega, \pi_z, \pi_r) = (0.5, -, +)$ and $(1.5, +, -)$ which belong to a spherical $1p_{3/2}$ shell, when the isotopes change from the prolate ground state to the oblate excited state. The level $(0.5, -, +)$ of proton and of neutron is lower than the level $(1.5, +, -)$ in the prolate ground states and the level $(1.5, +, -)$ is lower in the oblate excited states of Be isotopes. Thus two protons occupy the level $(0.5, -, +)$ in the prolate ground states making the nuclei prolate while they occupy the level $(1.5, +, -)$ in the oblate excited states making the nuclei oblate. Also shown in these simple calculations is that the protons are bound stronger in the isotope with more neutrons while neutron levels are somewhat insensitive to the number of neutrons and thus the nuclear size and also the neutron skin become larger as the neutron number increases. The binding energy of Be isotopes and the single neutron levels obtained here indicate the existence of the neutron magic number 6 instead of 8. The vanishing of the neutron magic number 8 was predicted in Refs.[11, 21, 22]. The behavior of the gap between the fourth and fifth neutron levels obtained here suggests that the disappearance of the magic number 8 is originated from a large deformation of nucleus. It is also shown that a level inversion between the fourth $(0.5, -, +)$ and the fifth $(0.5, +, +)$ levels occurs only in a largely deformed isotope, i.e., in ^8Be and ^{16}Be . This inversion is related to the intruder state $1/2^+$ in the ground state of ^{11}Be and ^9He [39, 40]. In these calculations for He and Be isotopes with Skyrme-IIIls interaction, no system with a clear indication of neutron halo was found. Instead of it we found $^8\text{He}+2n$, $2n+^8\text{He}+2n$, and $^{16}\text{Be}+2n$ like chain structures with clusters of two correlated neutrons for ^{10}He , ^{12}He , and ^{18}Be respectively. The correlated neutrons in the dineutron cluster have a positive single particle energy and are not bound to the ^8He or ^{16}Be cluster. These nuclei with chain structure may be either a resonance or one of Borromean nucleus [36]. More detailed investigations are needed. It is also shown that the nuclei ^8He and ^{14}Be are below the neutron drip line in which all nucleons are bound (with negative energy). We have also found that ^{16}Be in its ground state, as a single cluster without $2n$ correlated cluster, is beyond the neutron drip line with two neutrons in positive energy levels. Due to two neutrons in a positive energy level, ^{10}He and ^{16}Be have a negative two-neutron separation energy in agreement with data [32]. It is also shown that the neutron separation energy of ^6He is smaller than the separation energy of ^8He which is consistent with data [32]. The large rms radius of neutron distribution in ^6He , which is similar to the rms radius of ^8He , is originated from the large deformation of ^6He rather than from a halo structure. The most probable candidate of halo nucleus obtained here is the excited oblate state of ^{16}He which has a large neutron skin of 1.56 fm with a relatively small quadrupole moment.

We should notice here that the calculations done here cannot examine a nuclear structure with multi-centered clusters such as $^6\text{He}+^6\text{He}$ or $^8\text{He}+^4\text{He}$. The 2α cluster structure in ^{12}Be was found in Refs.[9, 10, 11] even though they missed the detailed nuclear structure. It was also shown, using relativistic mean field theory, that an excited state of ^{12}C is a chain structure of ^8Be and α clusters centered at two points while the ground state is oblate with a single center [6]. To study various states of these nuclei properly in a mean field approach, we need to extend our HF

calculation to allow nuclear structures with multi-centered clusters [6]. On the other hand to study halo structure we need to extend our calculation to allow highly excited single nucleon levels and to consider other nuclei such as C and O isotopes too. Notice here also that the binding energies obtained here with the Skyrme-IIIls interaction were overestimated with a large uncertainty due an approximate treatment of CM energy correction. But we believe that the main features of isotopes obtained here would not be altered when we use other mean field potentials with a proper treatment of CM energy correction. However, for a quantitative study in future, we need to implement CM energy correction more exactly and also need to investigate the dependence of the various behaviors of isotopes on the mean field potential used.

This work was supported by Grant No. KHU-20050313 of the Kyung Hee University Research Fund in 2005. The authors thank L. Zamick for helpful comments and discussions.

-
- [1] M. Seya, M. Kohno, and S. Nagata, Prog. Theor. Phys. **65**, 204 (1981).
 - [2] Y. Kanada-En'yo, H. Horiuchi, and A. Ono, Phys. Rev. **C52**, 628 (1995); Y. Kanada-En'yo and H. Horiuchi, Phys. Rev. **C52**, 647 (1995).
 - [3] W. von Oertzen, Z. Phys. **A354**, 37 (1996); **A357**, 355 (1997).
 - [4] N. Itagaki, S. Okabe, and K. Ikeda, Phys. Rev. **C62**, 034304 (1999).
 - [5] Y. Kanada-En'yo, Phys. Rev. **C66**, 011303 (2002).
 - [6] S.H. Hong and S.J. Lee, J. Korean Phys. Soc. **35**, 46 (1999).
 - [7] A.A. Korshennikov, et al., Phys. Lett. **B343**, 53 (1995).
 - [8] M. Freer, et al., Phys. Rev. Lett. **82**, 1383 (1999); Phys. Rev. **C63**, 034301 (2001).
 - [9] M. Ito, Y. Sakuragi, and Y. Hirabayashi, Phys. Rev. **C63**, 064303 (2001).
 - [10] G.F. Filippov, et al., Few Body Syst. **34**, 209 (2004); LANL preprint arXiv:nucl-th/0311063 (2003).
 - [11] Y. Kanada-En'yo and H. Horiuchi, Phys. Rev. **C68**, 014319 (2003).
 - [12] Akinobu Doté, Hisashi Horiuchi, and Yoshiko Kanada-En'yo, nucl-th/9705050 (1997).
 - [13] S. S. Chandel, S. K. Dhiman, and R. Shyam, Phys. Rev. **C68**, 054320 (2003).
 - [14] K.R. Sandhya Devi and S.E. Koonin, Phys. Rev. Lett. **47**, 27 (1981).
 - [15] N. Tajima, S. Takahara, and N. Onishi, Nucl. Phys. **A603**, 23 (1996).
 - [16] H. Liu, L. Zamick, and H. Jaqaman, Phys. Rev. **C31**, 2251 (1985).
 - [17] F.M. Marqués et al., Phys. Lett. **B476**, 219 (2000).
 - [18] P.G. Hansen and B. Jonson, Europhys. Lett. **4**, 409 (1987).
 - [19] L. Johannsen, A.S. Jensen, and P.G. Hansen, Phys. Lett. **B244**, 357 (1990).
 - [20] A.S. Jensen and K. Riisager, Phys. Lett. **B264**, 238 (1991).
 - [21] F.C. Barker, J. Phys. G, Nucl. Part. Phys. **2**, L45 (1976).
 - [22] H.T. Fortune, G.-B. Liu, and D.E. Alburger, Phys. Rev. **C50**, 1355 (1994).
 - [23] J. D. Walecka, Ann. Phys. (N.Y.) **83**, 491 (1974).
 - [24] S. J. Lee, J. Fink, A. B. Balantekin, M. R. Strayer, and A. S. Umar, P. -G. Reinhard, J. A. Maruhn and W. Greiner, Phys. Rev. Lett. **57**, 2916 (1986); **59**, 1171 (1987).
 - [25] C.E. Price and G.E. Walker, Phys. Rev. **C36**, 354 (1987).
 - [26] W. Pannert, P. Ring and J. Boguta, Phys. Rev. Lett. **59**, 2420 (1987).
 - [27] J.H. Lee, Y.J. Lee, and S.J. Lee, J. Korean Phys. Soc. **30**, 169 (1997).
 - [28] S.J. Lee, J. Korean Phys. Soc. **32**, 612 (1998).
 - [29] L. Zamick, et al., Annals of Phys. **212**, 402 (1991).
 - [30] S.J. Lee, et al., Phys. Lett. **196B**, 419 (1987).
 - [31] G. Gangopadhyay and Subinit Roy, J. Phys. **G31**, 1111 (2005).
 - [32] G. Audi, A.H. Wapstra and C. Thibault, Nucl. Phys. **A 729**, 337 (2003)
 - [33] J.W. Negele, Phys. Rev. **C1**, 1260 (1970).
 - [34] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. **A635**, 231 (1998).
 - [35] P.-G. Reinhard, D.J. Dean, W. Nazarewicz, J. Dobaczewski, J.A. Maruhn, and M.R. Strayer, Phys. Rev. **C60**, 014316 (1999).
 - [36] M.V. Zhukov, B.V. Danilin, D.V. Fedorov, J.M. Bang, I.J. Thompson, and J.S. Vaagen, Phys. Rep. **231**, 151 (1993).
 - [37] B. Jonson, Phys. Rep. **389**, 1 (2004).
 - [38] A. Ozawa, T. Kobayashi, T. Suzuki, K. Yoshida, and I. Tanihata, Phys. Rev. Lett. **84**, 5493 (2000).
 - [39] J. Al-Khalili and K. Arai, Phys. Rev. **C74**, 034312 (2006).
 - [40] T. Otsuka, et al., Phys. Rev. Lett. **87**, 082502 (2001).
 - [41] M.S. Fayache, E. Moya de Guerra, P. Sarriguren, Y.Y. Sharon, and L. Zamick, Phys. Rev. **C57**, 2351 (1998).